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Productivity analysis of horizontal wells intercepted by multiple finite-conductivity fractures

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Abstract: Horizontal wells in the anisotropic reservoirs can be stimulated by hydraulic fracturing in order to create multiple finite-conductivity vertical fractures. Several methods for evaluating the productivity of the horizontal wells have been presented in the literature. With such methods, however, it is still difficult to obtain an accurate result. This paper firstly presents the dimensionless conductivity theory of vertical fractures. Then models for calculating the equivalent wellbore radius and the skin factor due to flow convergence to the well bore are proposed after analyzing the steady-state flow in porous reservoirs. By applying the superposition principle to the pressure drop, a new method for evaluating the productivity of horizontal wells intercepted by multiple finite-conductivity fractures is developed. The influence of fracture conductivity and fracture half length on the horizontal well productivity is quantitatively analyzed with a synthetic case. Optimum fracture number and fracture space are further discussed in this study. The results prove that the method outlined here should be useful to design optimum fracturing of horizontal wells.

Key words: Production rate analysis, fractured horizontal wells, finite-conductivity vertical fractures, fracturing design optimization

1 Introduction

Compared with vertical wells, horizontal wells are suitable for thin reservoirs, low-permeability reservoirs, and heavy oil reservoirs. However, they fail for anisotropic reservoirs, where the vertical permeability is much less than the horizontal permeability. The advantage of horizontal wells diminishes with an increase in reservoir anisotropy. This situation can be improved by staged fracturing, which generates a number of vertical hydraulic fractures to intersect the horizontal wells, as shown in Fig. 1. Those intersected vertical fractures can reform the flow regimes near the well and increase the vertical permeability of the reservoir, thus enlarging the drainage area of the well. It has been proved that with multiple vertical hydraulic fractures, horizontal-well productivity can be increased significantly (Giger, 1986, 1987; Joshi, 1986; Soliman et al, 1988).

Current methods for productivity evaluation of horizontal wells have their drawbacks (Brown and Economides, 1992; Raghavan and Joshi, 1993; Lang et al, 1994; Raghavan et al, 1997; Xu et al, 2006; Zeng et al, 2007). With such methods, the fracture conductivity is either assumed to be infinite which leads to overestimate the productivity, or treated as finite in which a complicated Fredholm integral equation must be solved with boundary integral methods. In this paper, the equivalent wellbore radius and the skin due to flow convergence are proposed. By applying the superposition principle, a new analytical method is developed for evaluating the productivity of horizontal wells with multiple finite-conductivity vertical fractures.

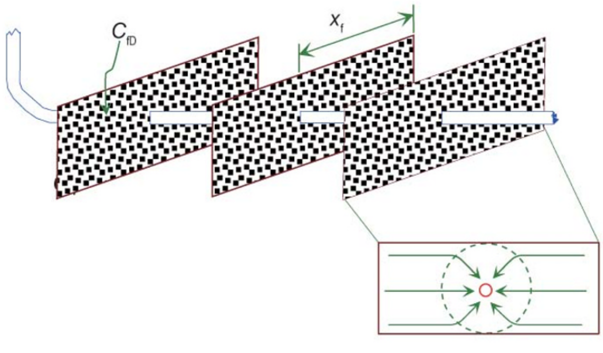

Fig. 1 Schematic of flow regimes in the fracture

2 Theory of fracture conductivity

In the process of hydraulic fracturing, sand proppants need to be injected into fractures or the fracture surface acidified in order to maintain the conductive channels after

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the fracturing pressure is removed. The fracture conductivity can be defined quantitatively with the hydraulic fracture conductivity, C_f, which is a multiplication product of fracture width (w) and fracture permeability (k_f), given by Prats et al (1960) as below:

$$C_f = k_f w \tag{1}$$

where C_f is the flow rate for unit viscosity fluid passing through a unit-height cross section of the fracture per unit pressure gradient; k_f is the permeability of the propped fracture, $10^{-3}\mu m^2$; and w_f is the average fracture width, m.

In reservoir engineering, with respect to hydraulic fracture conductivity, fractures can be divided into three types: infinite-conductivity fractures, uniform-flow fractures, and finite-conductivity fractures. Infinite-conductivity fractures imply that the conductivity is infinite and there is no pressure drop when the fluid flows along the fracture in any instant. Uniform-flow fractures imply that the flow rate along the unit cross section of the fracture is uniform. Finite-conductivity fractures consider that the conductivity is finite and the pressure drops when the fluid flows along the fracture but the flow rate along the unit cross section of the fracture is not uniform. Obviously, the finite-conductivity fracture is a representative type, which is closer to reality.

Theoretical analysis shows that there are two factors which influence productivity enhancement, namely, the dimensionless penetration ratio ($1/r_{eD}$) and the dimensionless fracture conductivity (C_{fD}), which can be expressed as follows:

$$r_{eD} = \frac{r_e}{x_f} \tag{2}$$

$$C_{fD} = \frac{k_f w_f}{k x_f} \tag{3}$$

where r_e is the reservoir drainage radius, m; x_f is the fracture half-length, m; and k is the reservoir permeability, $10^{-3}\mu m^2$.

For a given reservoir-fractured well system, the dimensionless fracture conductivity has much physical significance. Assuming that the fracture penetrates the reservoir formation completely so that the fracture height h_f is equal to the formation thickness h, the dimensionless conductivity can be reformatted as below:

$$C_{fD} = \frac{k_f w_f}{k x_f} \cdot \frac{h_f / \mu}{h / \mu} = \frac{w_f h_f (k_f / \mu)}{x_f h (k / \mu)} = \frac{q_{inf}}{q_{outf}} \tag{4}$$

where μ is the fluid viscosity, mPa·s.

From Darcy's law, q_{inf} is the inflow rate of fluids into the fracture per unit-pressure gradient, while q_{outf} is the outflow rate of fluids out from the fracture per unit-pressure gradient. Therefore, the fracture conductivity is actually a ratio of flow rate of fluids flowing in and out of the fracture per unit-pressure gradient. If the inflow matches the outflow, the fracture is deemed to be optimum. Considering the importance of fracture conductivity, it must be taken into account when conducting productivity analysis and fracture geometry optimization.

3 Equivalent wellbore radius

As we know, the flow performance in horizontal wells with multiple finite-conductivity fractures is complicated. Referring to the approaches of Prats et al (1960) and Cinco-Ley and Samaniego-V (1981), the fluid flow performance in fractures is treated as a combination of linear flow and radial flow, as shown in Fig. 1. After this, based on the general productivity formula of finite-conductivity vertical factures (Wang et al, 2004), a new method for analyzing the productivity of horizontal wells with multiple vertical fractures is developed by introducing equivalent wellbore radius together with skin due to the convergence of fluids into horizontal wellbore.

By solving the integral equation and asymptotic analysis, Wang et al (2004) presented a general productivity formula for wells with finite-conductivity vertical fractures, shown as below:

$$q_f = \frac{kh\left(P_{avg} - P_{wf}\right)}{1.842\times10^{-3}\mu B\left[\ln\frac{r_e}{2x_f} + \frac{3}{4} + f\left(C_{fD}\right)\right]} \tag{5}$$

with

$$f(C_{fD}) = \sum_{n=1}^{\infty}\frac{\pi C_{fD}}{n\left[2n + \pi C_{fD}(n+1)\right]} - \frac{\pi C_{fD}}{\pi C_{fD} + 2} \tag{6}$$

where q_f is the rate, m^3/d; k is the permeability, mD; h is the formation thickness, m; B is the volume factor, m^3/m^3; r_e is the drainage radius, m; $f(C_{fD})$ is a function of dimensionless conductivity.

The combination of Eq. (5) with the productivity equation for vertical wells producing at pseudo-steady-state gives:

$$\ln\frac{r_e}{r_{we}} - \frac{3}{4} = \ln\frac{r_e}{2x_f} + \frac{3}{4} + f\left(C_{fD}\right) \tag{7}$$

Rearranging Eq. (7) yields

$$r_{we} = 2x_f \exp\left\{-\left[\frac{3}{2} + f(C_{fD})\right]\right\} \tag{8}$$

where r_{we} is the equivalent wellbore radius of a well with finite-conductivity vertical fractures at the pseudo-steady state.

The computational results from Eq. (8) are consistent with the numerical results from the model proposed by Cinco-Ley and Samaniego-V (1981), as shown in Fig. 2. In the productivity analysis, a well with finite-conductivity vertical fractures can be equivalent to a general vertical well by introducing the equivalent wellbore radius model.

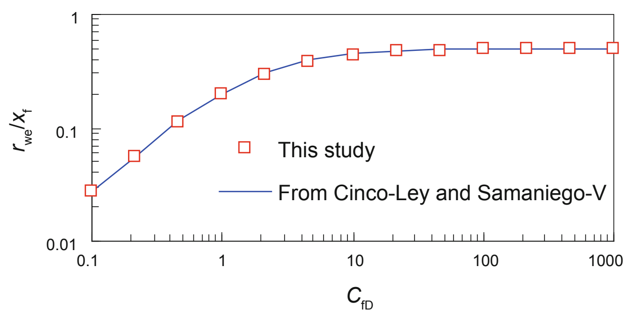

Fig. 2 A comparison of equivalent wellbore radii from different models

4 Skin factor due to flow convergence to the wellbore

As shown in Fig. 1, the fluid flow pattern in hydraulically fractured horizontal wells is special because the fluid flow is comprised of linear flow and radial flow, which is different from that in vertical wells. For vertical wells, the vertical fractures are in lateral contact with the wellbore so there is only linear flow in the fractures. Generally, Eq. (8) is only for a vertical well with vertical fractures. If it is applied to a horizontal well with vertical fractures, the additional pressure drop due to the convergence of fluids into the wellbore should be considered.

The steady-state pressure drop (ΔP_R) due to the radial flow in a medium with an outer radius of $h/2$ (half-reservoir height and a thickness of w (fracture width) can be written as below:

$$\Delta P_R = \frac{q\mu}{2\pi k_f w}\ln\frac{h}{2r_w} \quad (9)$$

where r_w is the wellbore radius, m; q is the rate.

The pressure drop ΔP_L due to the linear flow in the medium of a width of $h/2$, thickness of w, and length of h, can be written as below:

$$\Delta P_L = \frac{(q/2)\mu(h/2)}{k_f wh} \quad (10)$$

The combination of Eq. (9) with Eq. (10) gives the pressure-drop difference:

$$\Delta P_s = \Delta P_R - \Delta P_L = \frac{q\mu}{2\pi kh}\left[\frac{kh}{k_f w}\left(\ln\frac{h}{2r_w} - \frac{\pi}{2}\right)\right] = \frac{q\mu}{2\pi kh}\left[\frac{1}{C_{fD}}\frac{h}{x_f}\left(\ln\frac{h}{2r_w} - \frac{\pi}{2}\right)\right]$$

The skin factor due to the convergence of fluids is defined as (Brown and Economides, 1992):

$$S_c = \frac{1}{C_{fD}}\frac{h}{x_f}\left(\ln\frac{h}{2r_w} - \frac{\pi}{2}\right) \quad (11)$$

Then the equivalent wellbore radius of horizontal wells with a single vertical fracture can be expressed as:

$$r_{we} = 2x_f \exp\left\{-\left[\frac{3}{2} + f(C_{fD}) + S_c\right]\right\} \quad (12)$$

If the damage of fracture surface can be determined, the skin due to the damage can also be introduced into Eq. (12).

5 Productivity of horizontal wells intercepted by multiple finite-conductivity fractures

A horizontal well, located at the center of a circular reservoir, is completed with staged fracturing treatments. The drainage radius of the reservoir is r_e. The maximum half-length of the fracture is $x_{fmax,}$ and the distance between two neighboring fractures is d ($d > x_{fmax}$). Using the equivalent wellbore radius model and the pressure-drop superposition principle, the steady-state productivity of fractures can be calculated.

5.1 Horizontal wells with 5 fractures

A specific case presented by Raghavan and Joshi (1993) was considered, which shows a horizontal well with five infinite-conductivity fractures. As shown in Fig. 3, Fractures 1 and 5 have the longest half length x_{f1}, Fractures 2 and 4 have the second-longest half length x_{f2}, and Fracture 3 has the shortest half length x_{f3}. We further assume that q_1 is the production rate for Fractures 1 and 5, q_2 is the production rate for Fractures 2 and 4, and q_3 is the production rate for Fracture 3. If p_w is the bottom-hole flowing pressure, by applying the pressure-drop superposition to this specific system, we have

$$p_w - C = \frac{\mu B}{542.87kh}(q_1 \ln 4dr_{we1} + q_2 \ln 3d^2 + q_3 \ln 2d) \quad (13)$$

$$p_w - C = \frac{\mu B}{542.87kh}(q_1 \ln 3d^2 + q_2 \ln 2dr_{we2} + q_3 \ln d) \quad (14)$$

$$p_w - C = \frac{\mu B}{542.87kh}(q_1 \ln 4d^2 + q_2 \ln d^2 + q_3 \ln r_{we3}) \quad (15)$$

$$p_e - C = \frac{q_t \mu B}{542.87kh}\ln r_e \quad (16)$$

where r_{wej} (j =1, 2, 3) is the equivalent wellbore radius of Fracture j and its appropriate value can be obtained from Eq. (12). The total production rate is given by Raghavan and Joshi (1993):

$$q_t = \frac{542.87kh(p_e - p_w)}{\mu B} \Big/ \left[\ln\frac{r_e}{(4dr_{we1})^{q_{r1}}(3d^2)^{q_{r2}}(2d)^{q_{r3}}}\right] \quad (17)$$

where $q_{rj} = q_j / q_t$.

The ratios $q_{r1} : q_{r2} : q_{r3}$ is expressed as follows:

$$q_{r1} : q_{r2} : q_{r3} = (\ln 2 \ln 3 - \ln\frac{2r_{we2}}{3d}\ln\frac{r_{we3}}{2d}) : (\ln\frac{r_{we3}}{2d}\ln\frac{3d}{4r_{we1}} + \ln\frac{d}{r_{we1}}\ln 2) : (\ln\frac{d}{r_{we1}}\ln\frac{2r_{we2}}{3d} + \ln 3 \ln\frac{3d}{4r_{we1}}) \quad (18)$$

The equations above can be easily calculated. Roberts et al (1991) recommend that the minimum distance, d, between fractures be greater than $2x_f$. Then the correlations between

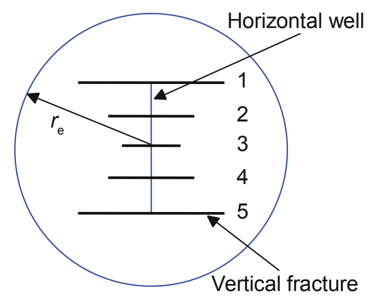

Fig. 3 Schematic of the fractured horizontal well (after Raghavan and Joshi, 1993)

total flow rate and fractional flow rate can be obtained from Eq. (18). Similarly, the method we used above can be easily applied to other complicated situations.

5.2 Horizontal well with *N* fractures

We use q_i to denote the production rate of Fracture i, d_{ji} to denote the distance between Fracture j and Fracture i, and r_{wei} to denote the equivalent wellbore radius of Fracture i. By applying the pressure-drop superposition to the well with N fractures, we have

$$\begin{cases} p_w - C = \dfrac{\mu B}{542.87kh} \times \left(q_1 \ln r_{we1} + q_2 \ln d_{21} + q_3 \ln d_{31} + \cdots + q_N \ln d_{N1}\right) \\ p_w - C = \dfrac{\mu B}{542.87kh} \times \left(q_1 \ln d_{12} + q_2 \ln r_{we2} + q_3 \ln d_{32} + \cdots + q_N \ln d_{N2}\right) \\ p_w - C = \dfrac{\mu B}{542.87kh} \times \left(q_1 \ln d_{13} + q_2 \ln d_{23} + q_3 \ln r_{we3} + \cdots + q_N \ln d_{N3}\right) \\ \vdots \\ p_e - C = \dfrac{\mu B}{542.87kh} \times \left(q_1 \ln r_e + q_2 \ln r_e + q_3 \ln r_e + \cdots + q_N \ln r_e\right) \end{cases} \tag{19}$$

The total production rate q_t of the well is:

$$q_t = q_1 + q_2 + \cdots + q_N \tag{20}$$

By introducing the dimensionless productivity index J_i, equations above are rearranged as follows:

$$\begin{cases} 0 = J_1 \ln \dfrac{r_{we1}}{d_{12}} + J_2 \ln \dfrac{d_{21}}{r_{we2}} + J_3 \ln \dfrac{d_{31}}{d_{32}} + \cdots + J_N \ln \dfrac{d_{N1}}{d_{N2}} \\ 0 = J_1 \ln \dfrac{r_{we1}}{d_{13}} + J_2 \ln \dfrac{d_{21}}{d_{23}} + J_3 \ln \dfrac{d_{31}}{r_{we3}} + \cdots + J_N \ln \dfrac{d_{N1}}{d_{N3}} \\ 0 = J_1 \ln \dfrac{r_{we1}}{d_{14}} + J_2 \ln \dfrac{d_{21}}{d_{24}} + J_3 \ln \dfrac{d_{31}}{d_{34}} + \cdots + J_N \ln \dfrac{d_{N1}}{d_{N4}} \\ \vdots \\ 1 = J_1 \ln \dfrac{r_e}{r_{we1}} + J_2 \ln \dfrac{r_e}{d_{21}} + J_3 \ln \dfrac{r_e}{d_{31}} + \cdots + J_N \ln \dfrac{r_e}{d_{N1}} \end{cases} \tag{21}$$

where the dimensionless productivity index is defined as

$$J_i = \frac{q_i \mu B}{542.87kh(p_e - p_w)} \qquad i = 1, 2, \cdots, N \tag{22}$$

The dimensionless productivity index J_i of each fracture is obtained by solving equations above, and then the production rate of each fracture is determined.

5.3 Case study

For this case, a horizontal well is stimulated by four vertical fractures. The well is drilled at the center of a homogeneous reservoir. Firstly, the influence of the half length and dimensionless conductivity on the well productivity is presented. Then, the influence of the fracture number on the total well production-rate and each fracture production-rate is tested.

Essential parameters for this case are available: μ = 20.0 mPa·s, r_w = 0.1 m, B = 1.1, k = 10 mD, $2L$ = 400.0 m, h = 10.0 m, z_w = 5.0 m, r_e = 350.0 m (z_w is the well location in the z direction, L is the horizontal-well half length). By solving Eq. (22) presented above, the results for this case are obtained, as shown in Fig. 4 through Fig. 6.

In this case, of which four fractures have identical length and uniform distribution, shown in Fig. 4 and Fig. 5, the outermost fractures have higher productivity indexes due to the larger drainage area. The inner fractures have lower productivity indexes due to the interference from outer fractures. Notice that the larger the available drainage area, the more sufficient the fluid flow from the formation into the fracture, and the greater the well production will be. The influence of the half length and dimensionless conductivity of the fracture on outer fracture productivity is bigger than that on inner ones. So in order to achieve a uniform production profile, one can first adjust the facture length, and then adjust the dimensionless fracture conductivity and fracture space.

As shown in Fig. 6, with the increase in the fracture number, the well production rate increases rapidly at the beginning and almost stabilizes later. The output of each fracture decreases as the fracture number increases. Based on

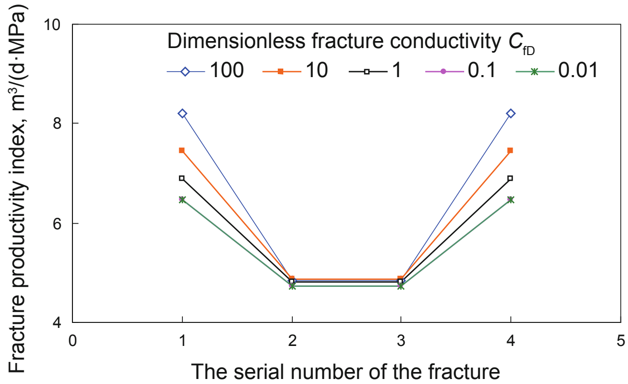

Fig. 4 Curves of productivity index varying with C_{fD}

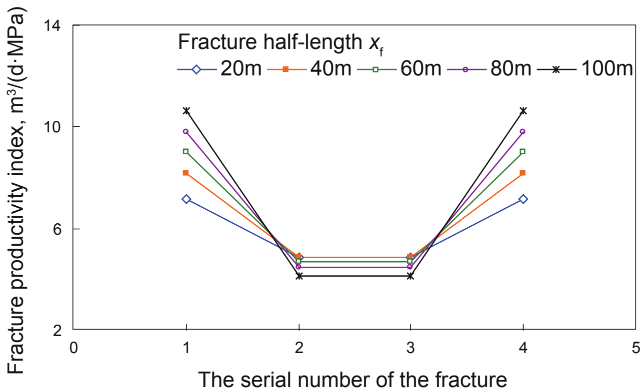

Fig. 5 Curves of productivity index varying with x_f values

this result, the optimum fracture number can be obtained with the combination of production-rate trends with the economic conditions. In this case, the optimum fracture number is five, as marked in Fig. 6.

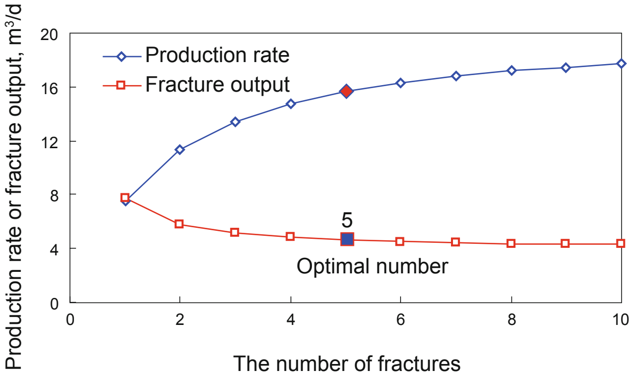

Fig. 6 Production rate and fracture output varying with fracture number

In addition, we notice that the effect of flow convergence can be negligible unless the dimensionless fracture conductivity is very small or the half length is very short.

6 Conclusions

1) Dimensionless fracture conductivity is the key parameter which affects the fracture efficiency. It is a ratio of the inflow to the outflow of the fluid in the fracture under unit-pressure gradient. Based on this value, the fracture geometry can be optimally designed.

2) An equivalent wellbore radius is obtained by combining productivity formulas for fractured wells and normal vertical wells in accordance with the production-equivalent principle. The accuracy of this model is proved and the significant influence of dimensionless fracture conductivity on well productivity is further verified.

3) The typical flow pattern of horizontal wells within a fracture is radial flow followed by linear flow. Based on this theory, the skin due to flow convergence is introduced and thus a new method for evaluating the productivity of horizontal wells with multiple finite-conductivity vertical fractures is developed. It has been proved that the skin due to flow convergence normally has a negligible effect on the well productivity.

4) It has been proved that there is mutual interference among the fractures. For the fractures with identical length and proportional space, the outermost fractures contribute more than the middle ones to enhance the production. The total production rate of the well increases as the fracture number increases, but the production rate of each fracture decreases. Optimization design can be operated by combining production rate trends and economic costs.

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